An Optimal Mechanism for Sponsored Search Auction

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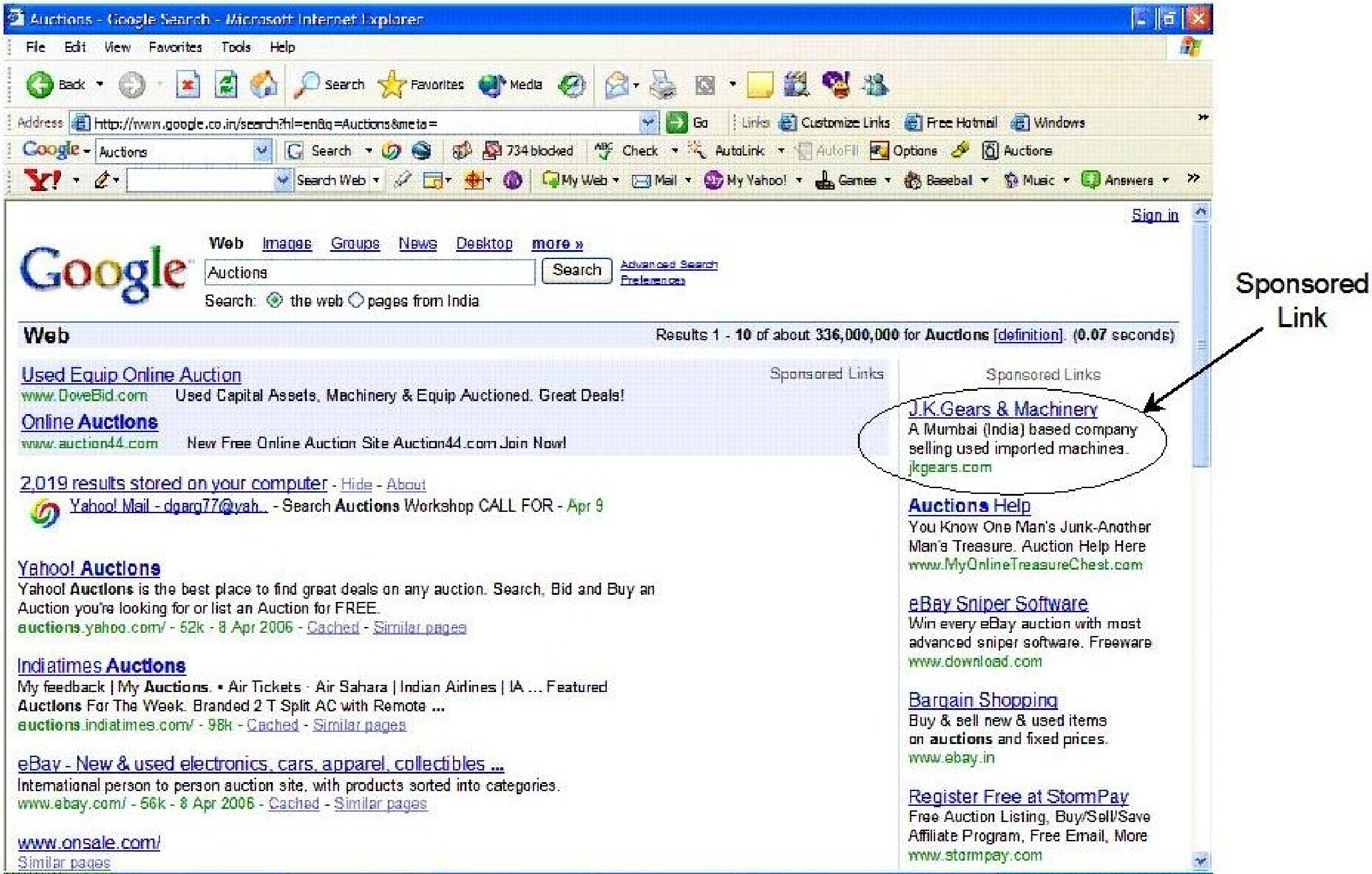
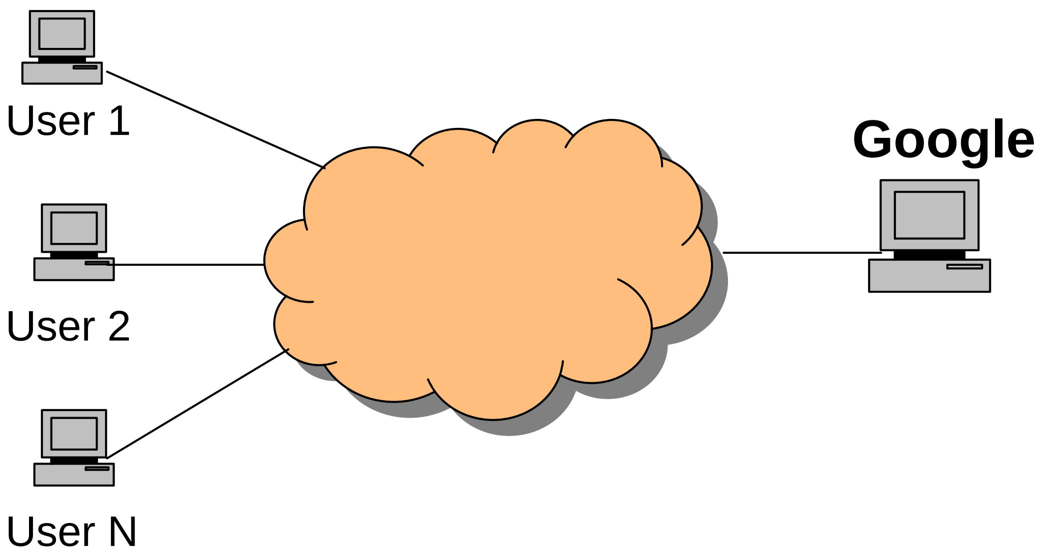
Introduction

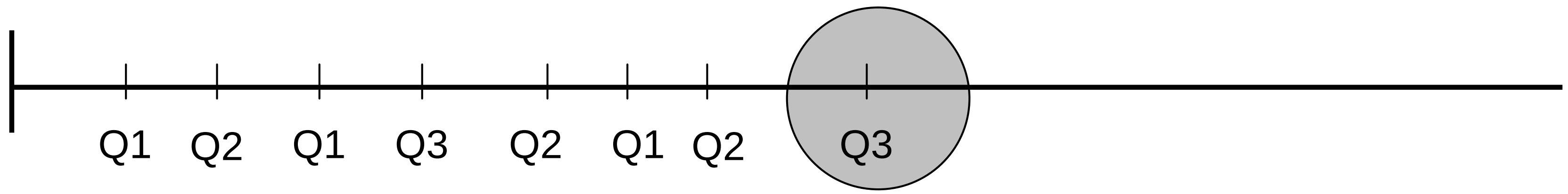
- **Problem Definition**



Introduction

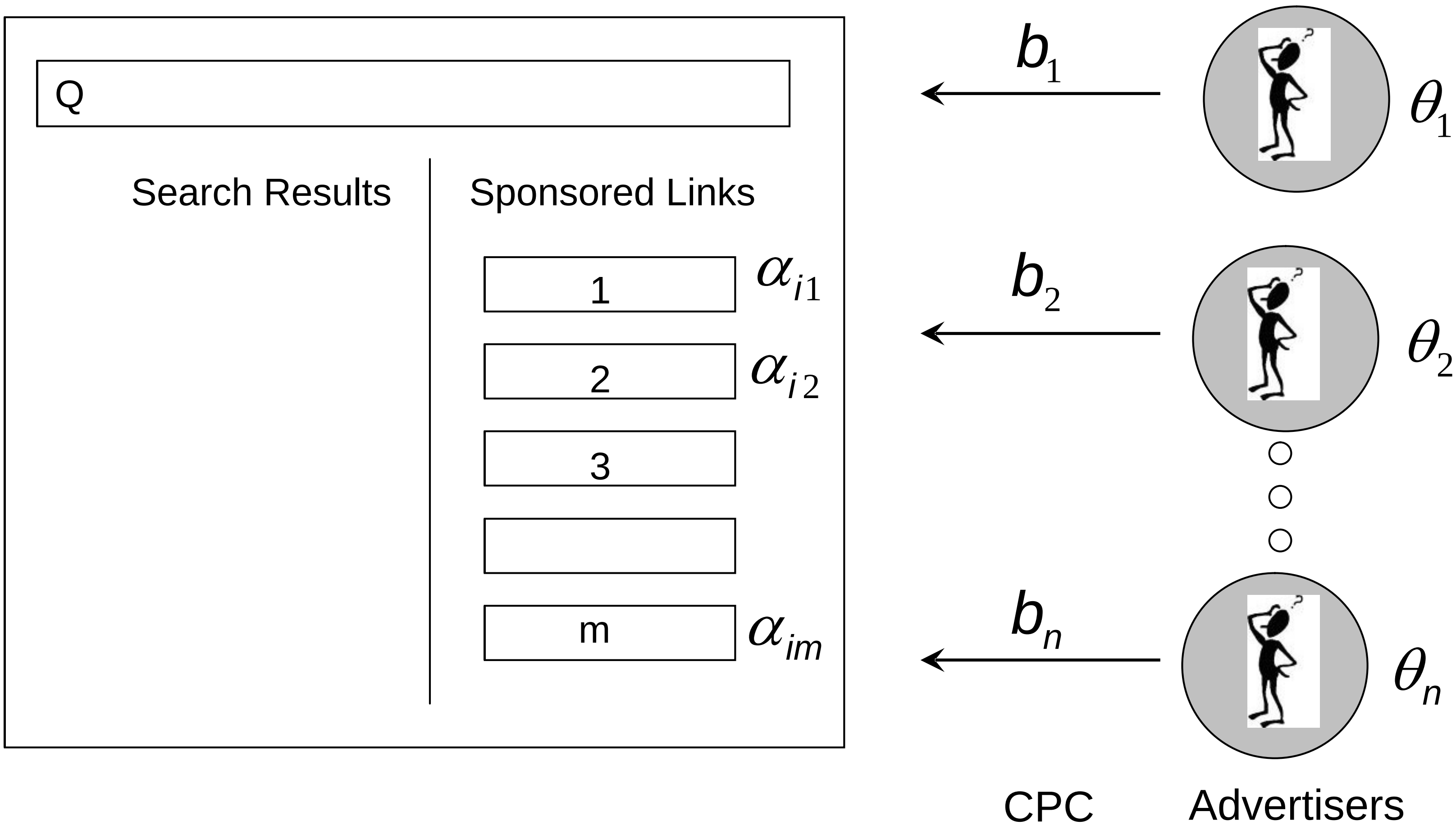
- **Problem Definition: Sequence of Queries**





Introduction

- **Problem Definition: Bids, Valuations, and Click Probabilities**



Introduction

- **Problem Definition: Bids, Valuations, and Click Probabilities**

$b = (b_1, \cdots, b_n) =$ Bid vector of advertisers

$b^{(1)}, \cdots, b^{(n)} =$ Decreasing ordering of the bids

$\theta_i =$ Value derived out of a click by advertiser i

$\quad = $ Type of advertiser i

$\Theta_i =$ Set of types of advertiser i

$\theta = (\theta_1, \cdots, \theta_n) =$ Type vector of advertisers

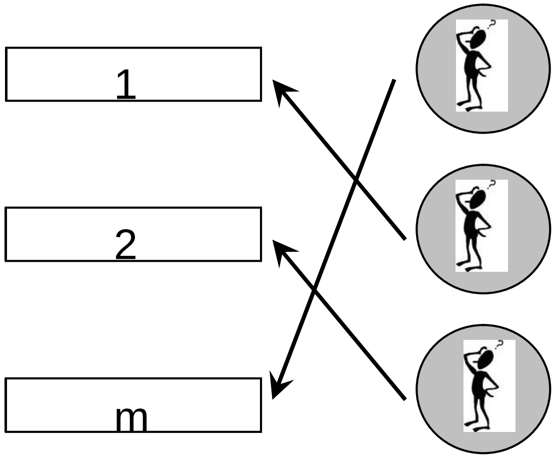
$\alpha_{ij} =$ Click probability of i^{th} Ad in j^{th} position

$1 \geq \alpha_{i1} \geq \alpha_{i2} \geq \cdots \geq \alpha_{im} \geq 0 \; \forall i \in N$ (AAE Assumption)

Introduction

- **Problem Definition: Search Engine's Problem**

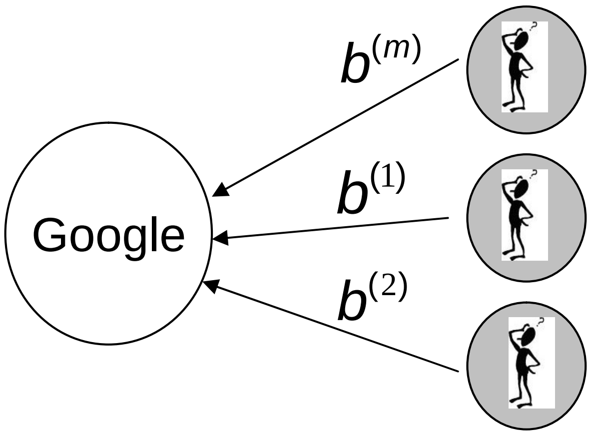
- **Allocation Rule**



Who should be allocated what ?

$$y_{ij}(b) = \begin{cases} 1 & \text{if advertiser } i \text{ is allocated slot } j \\ 0 & \text{o/w} \end{cases}$$

- **Payment Rule**



Which advertiser should be charged what price ?

$p_i(b) =$ Price that is charged from advertiser i for per click

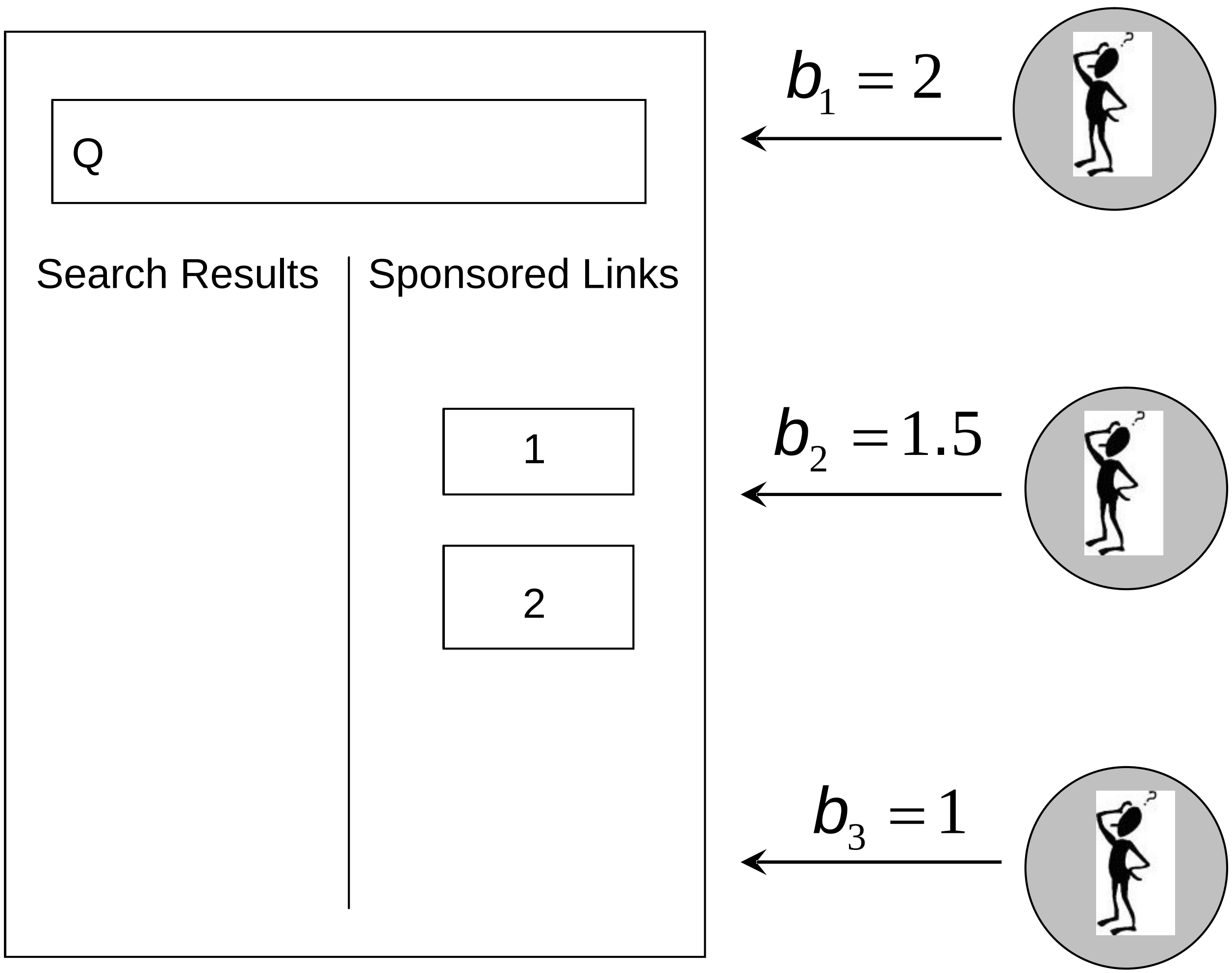
Introduction

- **Recent Literature**
  - B. Edelman, M. Ostrovsky, and M. Schwarz, "Internet Advertising and the Generalized Second Price Auction: Selling Billions of Dollars Worth of Keywords", *Mimeo,* September, 2005
  - J. Feng, "Optimal Mechanism for selling a set of Commonly Ranked Objects", *Mimeo*, *February* 2005
  - S. Lahaie, "An Analysis of Alternative Slot Auction Designs for Sponsored Search", *ACM Conference on Electronic Commerce (EC'06)*, *Ann Arbor, MI,* June 11-15, 2006
  - G. Aggarwal, A. Goel, and R. Motwani, "Truthful Auction for Pricing Search Keywords", *ACM Conference on Electronic Commerce (EC'06)*, *Ann Arbor, MI,* June 11-15, 2006
  - H. R. Varaian, "Position Auctions", *Mimeo*, February 2006

Outline

- ✓ Introduction
  - ✓ Problem Definition
  - ✓ Significance
  - ✓ Recent Literature
- Three well known mechanisms
  - Generalized First Price (GFP)
  - Generalized Second Price (GSP)
  - Vickrey-Clarke-Groves (VCG)
- A new mechanism – Optimal (OPT) Mechanism
- What is the best mechanism for Sponsored Search Auction?
- Comparison of OPT with GSP and VCG
  - Incentive Compatibility
  - Expected Revenue of the Search Engine
  - Individual Rationality
  - Computational Complexity

Generalized First Price (GFP)



$y_{11}(b) = 1$

$y_{12}(b) = 0$

$p_1(b) = 2$

$y_{21}(b) = 0$

$y_{22}(b) = 1$

$p_2(b) = 1.5$

$y_{31}(b) = 0$

$y_{32}(b) = 0$

$p_3(b) = 0$

$$b = (2, 1.5, 1)$$

Generalized First Price (GFP)

- **Allocation Rule**

Allocate the slots in decreasing order of bids

$$y_{ij}(b) = \begin{cases} 1 & \text{if } b_i = b^{(j)} \text{ and } j \leq \min(m,n) \\ 0 & \text{o/w} \end{cases}$$

- **Payment Rule**

For every user click, charge the advertiser his bid

$$p_i(b) = \begin{cases} b_i & \text{if advertiser } i\text{'s Ad is displayed} \\ 0 & \text{o/w} \end{cases}$$

✍ *Introduced by Overture in 1997*

Generalized Second Price (GSP)

- **Allocation Rule**

  - ***Yahoo* Rule**

    Allocate the slots in decreasing order of bids

  - **Greedy Rule**

    Allocate 1st slot to advertiser $i_1 = \underset{i \in N}{\arg\max}(\alpha_{i1} b_i)$

    Allocate 2nd slot to advertiser $i_2 = \underset{i \in N \setminus i_1}{\arg\max}(\alpha_{i2} b_i)$

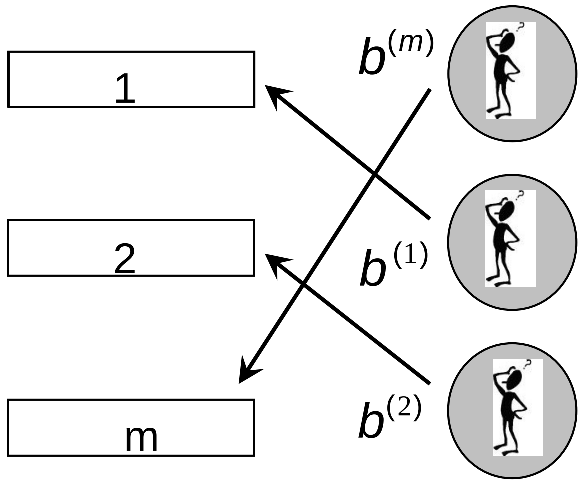
    ○

    ○

  - ***Google* Rule**

    Allocate the slots in decreasing order of Ranking Score

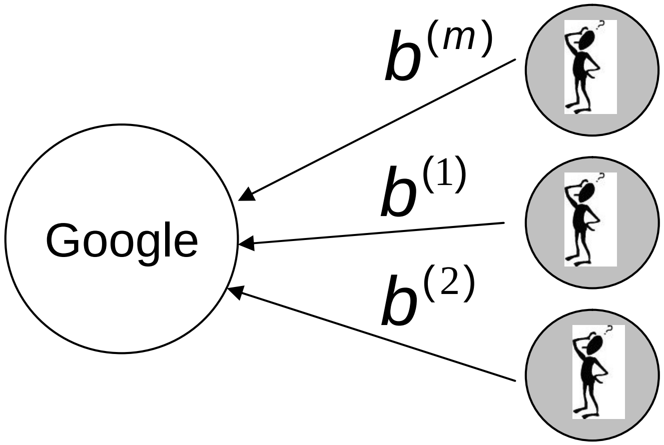
    Ranking Score = $b_i \times CTR_i$



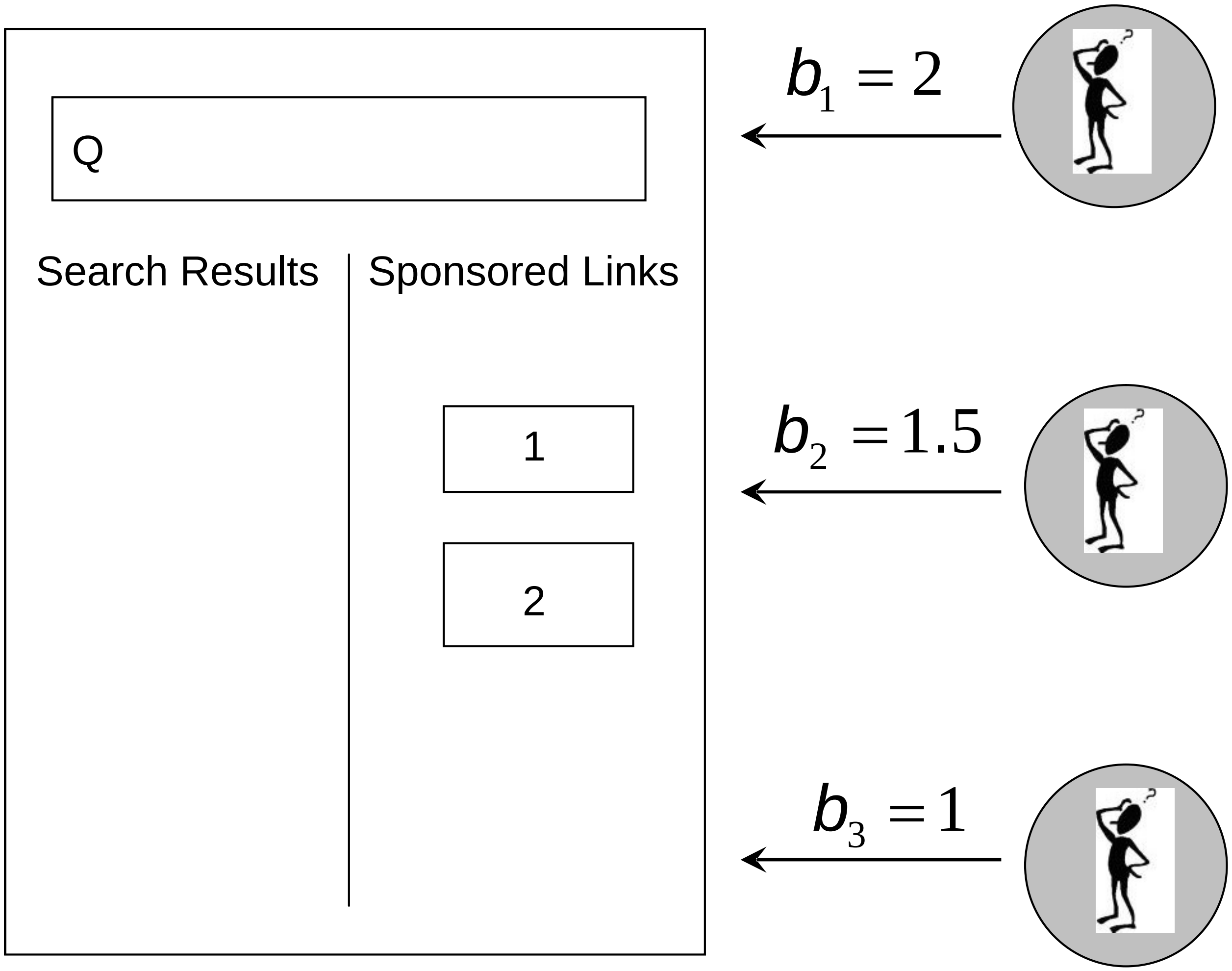
Introduced by Google in 2002 (Above facts are based on literature)

Generalized Second Price (GSP)

- **Payment Rule**
  - For every click, charge next highest bid + $0.01
  - The bottom most advertiser is charged highest disqualified bid +$0.01
  - charge 0 if no such bid



Generalized Second Price (GSP)



$y_{11}(b) = 1$
$y_{12}(b) = 0$
$p_1(b) = 1.5$

$y_{21}(b) = 0$
$y_{22}(b) = 1$
$p_2(b) = 1$

$y_{31}(b) = 0$
$y_{32}(b) = 0$
$p_3(b) = 0$

$$b = (2, 1.5, 1)$$

Generalized Second Price (GSP)

- **Allocation Rule**

Greedy

$$\begin{bmatrix} \alpha_{11} & \cdots & \alpha_{1m} \\ \vdots & \ddots & \vdots \\ \alpha_{n1} & \cdots & \alpha_{nm} \end{bmatrix} \begin{bmatrix} b_1 \\ \vdots \\ b_n \end{bmatrix} \begin{bmatrix} CTR_1 \\ \vdots \\ CTR_n \end{bmatrix}$$

Yahoo

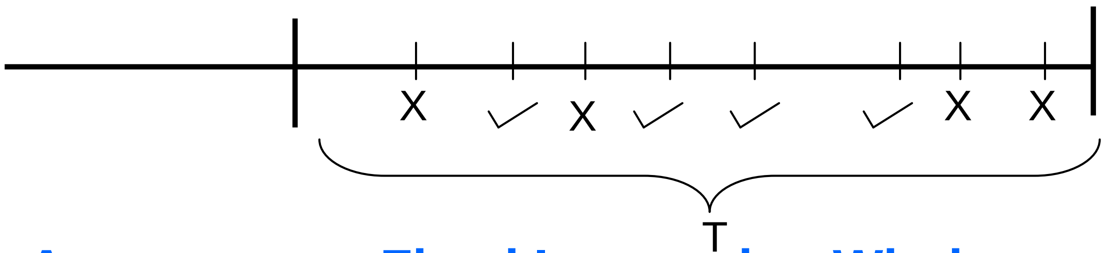
Google

$$CTR_i = \sum_{j=1}^{m} y_{ij}\alpha_{ij} \quad \Rightarrow \quad CTR_i \leq \sum_{j=1}^{m} \alpha_{ij}$$

Generalized Second Price (GSP)

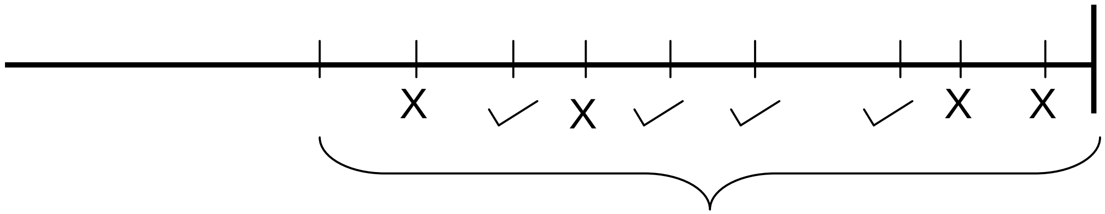
- **Learning CTR and Click Probabilities**

- **Average over Fixed Time Window**



$$CTR_i = \frac{C_i}{I_i}; \alpha_{ij} = \frac{C_{ij}}{I_{ij}}$$

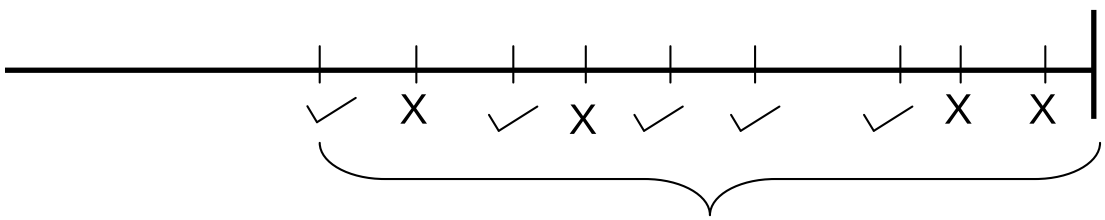
- **Average over Fixed Impression Window**



$$CTR_i = \frac{C_i}{1000}; \alpha_{ij} = \frac{C_{ij}}{1000}$$

$$I_i = 1000$$

- **Average over Fixed Click Window**



$$CTR_i = \frac{100}{I_i}; \alpha_{ij} = \frac{100}{I_{ij}}$$

$$C_i = 100$$

Generalized Second Price (GSP)

Relationship Among Allocation Rules

(AE) Problem

Max

$$\sum_{i=1}^{n} b_i \left(\sum_{j=1}^{m} \alpha_{ij} y_{ij}(b) \right)$$

$$= \sum_{i=1}^{n} b_i v_i \left(y(b) \right)$$

s.t.

$$\sum_{i=1}^{n} y_{ij}(b) \le 1 \; \forall j \in M$$

$$\sum_{j=1}^{m} y_{ij}(b) \le 1 \; \forall i \in N$$

$$0 \le y_{ij} \; \forall i \in N, \forall j \in M$$

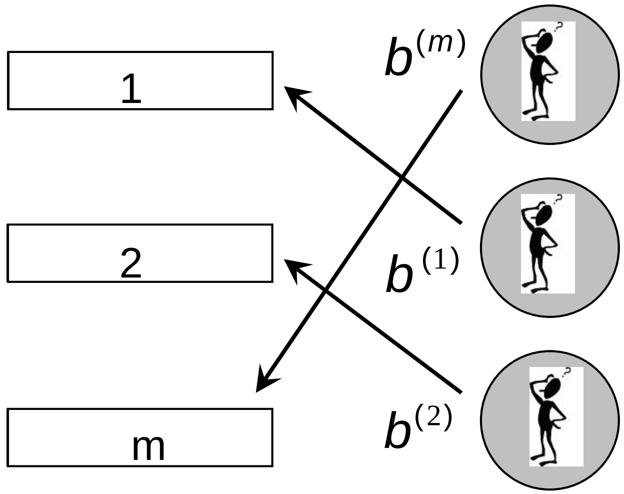
Proposition

Let click probabilities satisfy AAE assumption

- Greedy allocation rule is an optimal solution of the (AE) Problem
- If click probabilities depend only on identity of the advertiser and are independent of the position of the Ad, i.e. $\alpha_{i1} = \alpha_{i2} = \cdots = \alpha_{im} = CTR_i$ then greedy rule and Google rule result in the same allocation
- If click probabilities depend only on position of the Ad and are independent of the identity of the advertiser, i.e. $\alpha_{1j} = \alpha_{2j} = \cdots = \alpha_{nj} = \alpha_j$ then greedy rule and Yahoo! rule result in the same allocation

Vickrey-Clarke-Groves (VCG)

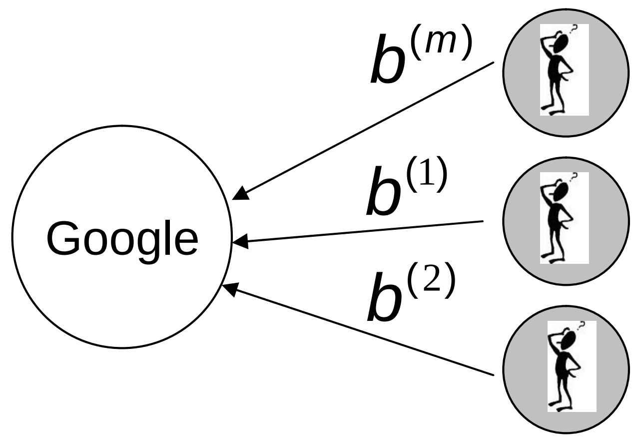
- **Allocation Rule**
  - Solution of (AE) Problem
  - Same as Yahoo! allocation under the assumption that click probability depends only on position



- **Payment Rule**

$$t_i(b) = \left[\sum_{j \neq i} b_j v_j(y^*_{-i}(b))\right] - \left[\sum_{j \neq i} b_j v_j(y^*(b))\right]$$

$$p^{(j)}(b) = \frac{t^{(j)}(b)}{\alpha_j}$$



Vickrey-Clarke-Groves (VCG)

- **Payment Rule**

<u>*Case 1*</u> $(m < n)$

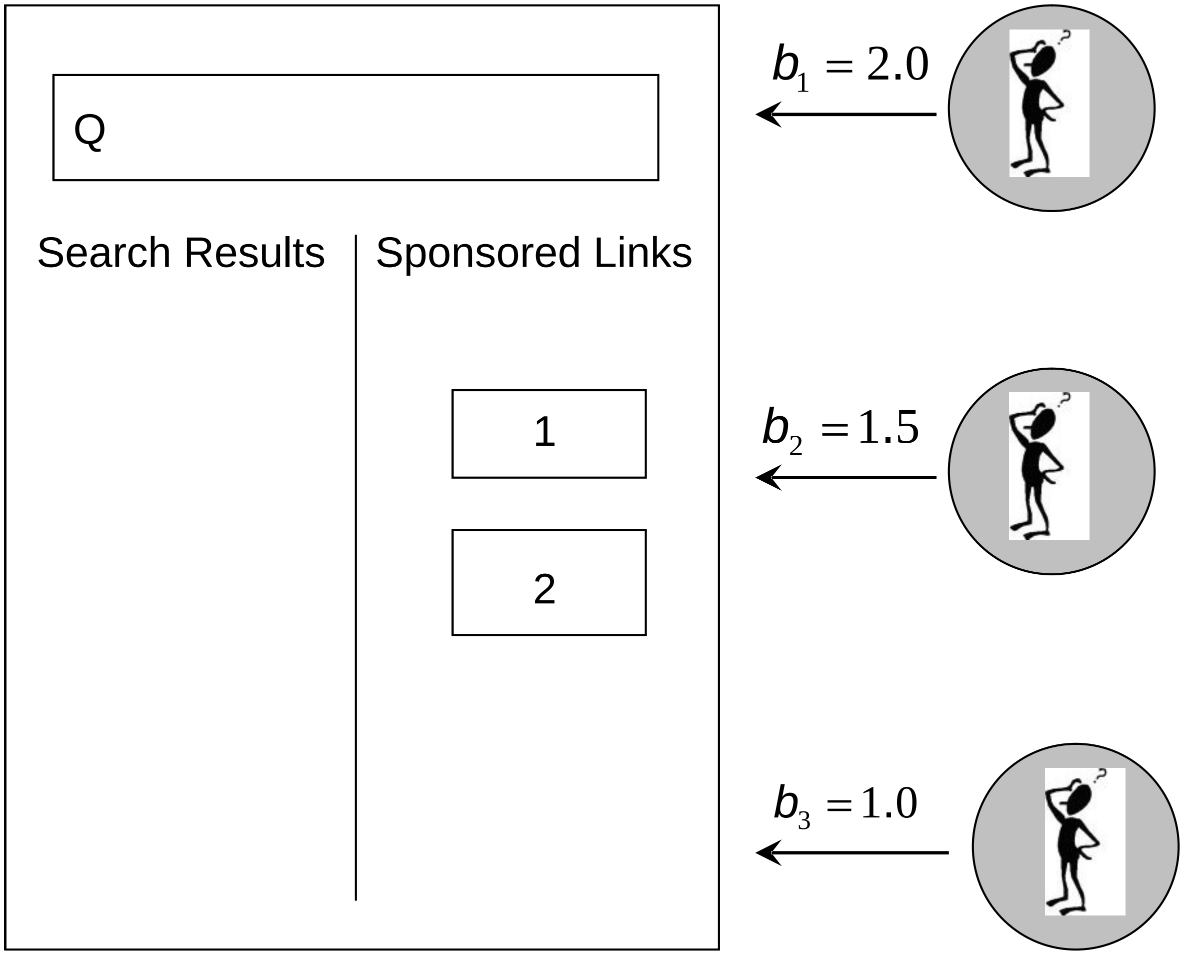
$$p^{(j)}(b) = \begin{cases} \frac{1}{\alpha_j}\left[\sum_{k=j}^{m-1} \beta_k b^{(k+1)}\right] + \frac{\alpha_m}{\alpha_j} b^{(m+1)} & \text{if } 1 \le j \le (m-1) \\ b^{(m+1)} & \text{if } j = m \\ 0 & \text{if } m < j \le n \end{cases}$$

<u>*Case 2*</u> $(n \le m)$

$$p^{(j)}(b) = \begin{cases} \frac{1}{\alpha_j}\left[\sum_{k=j}^{n-1} \beta_k b^{(k+1)}\right] & \text{if } 1 \le j \le (n-1) \\ 0 & \text{if } j = n \end{cases}$$

where $\beta_k = (\alpha_k - \alpha_{k+1})$

Vickrey-Clarke-Groves (VCG)



$b_1 = 2.0$

$y_{11}(b) = 1$

$y_{12}(b) = 0$

$p_1(b) = 1.5\left(1 - \frac{\alpha_2}{3\alpha_1}\right)$

$b_2 = 1.5$

$y_{21}(b) = 0$

$y_{22}(b) = 1$

$p_2(b) = 1$

$b_3 = 1.0$

$y_{31}(b) = 0$

$y_{32}(b) = 0$

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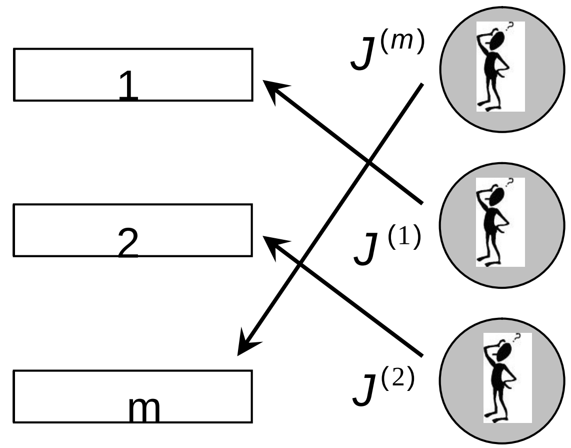
Optimal (OPT)

- **Allocation Rule**



$$y_{ij}(b) = \begin{cases} 0 & \forall 1 \le j \le n \quad : \quad \text{if } J_i(b_i) < 0 \\ 1 & \forall 1 \le j \le m \quad : \quad \text{if } J_i(b_i) = J^{(j)} \\ 0 & \forall m < j \le n \quad : \quad \text{if } J_i(b_i) = J^{(j)} \end{cases}$$

where $J^{(j)}$ is the j^{th} highest value among $J_i(b_i) = \left(b_i - \frac{1 - \Phi_i(b_i)}{\phi_i(b_i)} \right)$

(Assumption: $J_i(b_i)$ is non decreasing: True for Uniform, Exponential)

- **Proposition**

- Advertisers are symmetric, i.e.

$$\Theta_1 = \Theta_2 = \cdots = \Theta_n = \Theta$$
$$\Phi_1(.) = \Phi_2(.) = \cdots = \Phi_n(.)$$

- $J_i(.) > 0 \ \forall i = 1, \cdots, n$

$\Rightarrow$

For a given bid vector b, the OPT results in the same allocation as the GSP and the VCG, i.e. allocate in decreasing order of bids

Optimal (OPT)

- **Payment Rule**

Case 1 $(m < n)$

$$p_i(b_i, b_{-i}) = \begin{cases} \frac{1}{\alpha_r}\left[\sum_{k=r}^{m-1} \beta_k z_{ik}(b_{-i})\right] + \frac{\alpha_m}{\alpha_r} z_{im}(b_{-i}) & \text{if } 1 \le r \le (m-1) \\ z_{im}(b_{-i}) & \text{if r} = m \\ 0 & \text{o/w} \end{cases}$$

Case 2 $(n \le m)$

$$p_i(b_i, b_{-i}) = \begin{cases} \frac{1}{\alpha_r}\left[\sum_{k=r}^{n-1} \beta_k z_{ik}(b_{-i})\right] + \frac{\alpha_n}{\alpha_r} z_{in}(b_{-i}) & \text{if } 1 \le r \le (n-1) \\ z_{in}(b_{-i}) & \text{if r} = n \\ 0 & \text{o/w} \end{cases}$$

where

- r is the position at which advertiser i is allocated
- $\beta_k = (\alpha_k - \alpha_{k+1})$
- $z_{ij}(b_{-i})$ is the minimum bid for the advertiser i which can make him win j^{th} slot against the bid vector b_{-i} from other advertisers

Optimal (OPT)

- **Payment Rule when Advertisers are Symmetric**

$$\Theta_1 = \Theta_2 = \cdots = \Theta_n = \Theta = [L, U]$$

$$\Phi_1(.) = \Phi_2(.) = \cdots = \Phi_n(.)$$

Case 1 $(m < n)$

$$p_i(b_i, b_{-i}) = \begin{cases} \frac{1}{\alpha_r}\left[\sum_{k=r}^{m-1} \beta_k b^{(k+1)}\right] + \frac{\alpha_m}{\alpha_r} b^{(m+1)} & \text{if } 1 \le j \le (m-1) \\ b^{(m+1)} & \text{if } j = m \\ 0 & \text{if } m < j \le n \end{cases}$$

Case 2 $(n \le m)$

$$p_i(b_i, b_{-i}) = \begin{cases} \frac{1}{\alpha_r}\left[\sum_{k=r}^{n-1} \beta_k b^{(k+1)}\right] + \frac{\alpha_n}{\alpha_r} L & \text{if } 1 \le j \le (n-1) \\ L & \text{if } j = n \end{cases}$$

Optimal (OPT)

- **Proposition**

- Advertisers are symmetric, i.e.
  $\Theta_1 = \Theta_2 = \cdots = \Theta_n = \Theta = [L, U]$
  $\Phi_1(.) = \Phi_2(.) = \cdots = \Phi_n(.)$
- $J_i(.) > 0 \; \forall i = 1, \cdots, n$
- m < n

$\Rightarrow$

Payment Rule

OPT $\equiv$ VCG

- Advertisers are symmetric, i.e.
  $\Theta_1 = \Theta_2 = \cdots = \Theta_n = \Theta$
  $\Phi_1(.) = \Phi_2(.) = \cdots = \Phi_n(.)$
- $J_i(.) > 0 \; \forall i = 1, \cdots, n$
- m = n

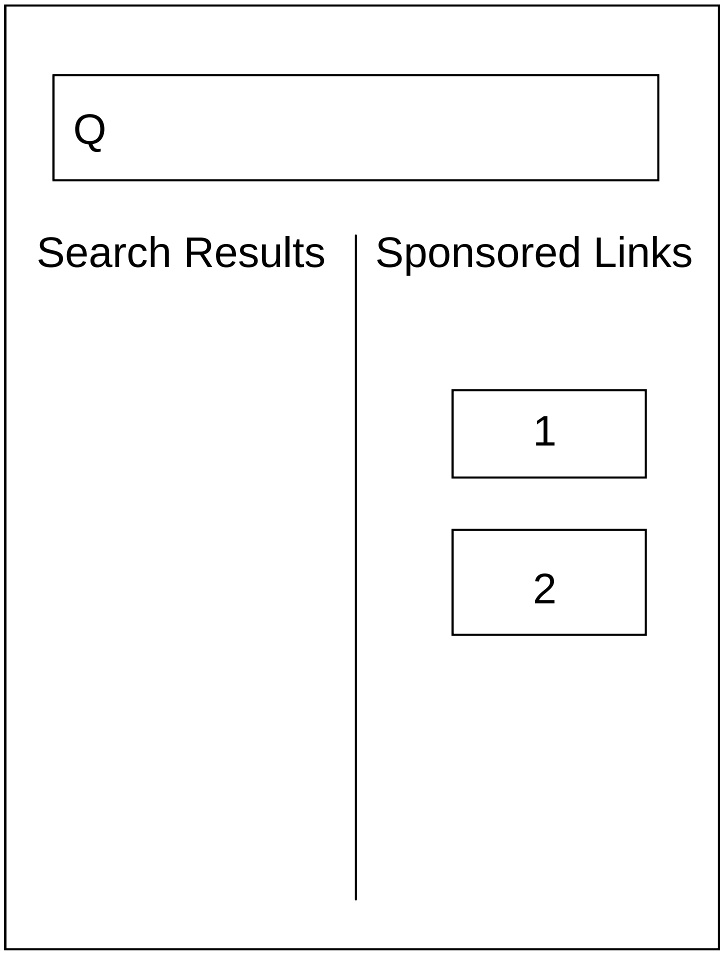
$\Rightarrow$

Payment Rule

OPT $\equiv$ VCG

(up to a constant factor L)

Example: OPT



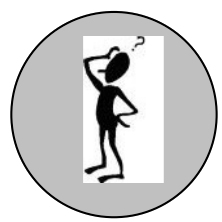
$\theta_1 = 2.0$

$$\Theta_1 = [1,2]$$
$$\Phi_1(x) = (x-1); \phi_1(x) = 1$$
$$J_1(2) = 2 - \frac{1-1}{1} = 2$$
$$y_{11}(b) = 1; p_1(b) = 1.5\left(1 - \frac{\alpha_2}{3\alpha_1}\right)$$

$\theta_2 = 1.5$

$$\Theta_2 = [1,2];$$
$$\Phi_2(x) = (x-1); \phi_2(x) = 1$$
$$J_2(1.5) = 1.5 - \frac{1-0.5}{1} = 1$$
$$y_{22}(b) = 1; p_2(b) = 1$$

$\theta_3 = 1.0$



$$\Theta_3 = [1,2];$$
$$\Phi_3(x) = (x-1); \phi_3(x) = 1$$
$$J_3(1) = 1 - \frac{1-0}{1} = 0$$
$$y_{3j} = 0; p_3(b) = 0$$

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What is the best Mechanism for Sponsored Search Auction?

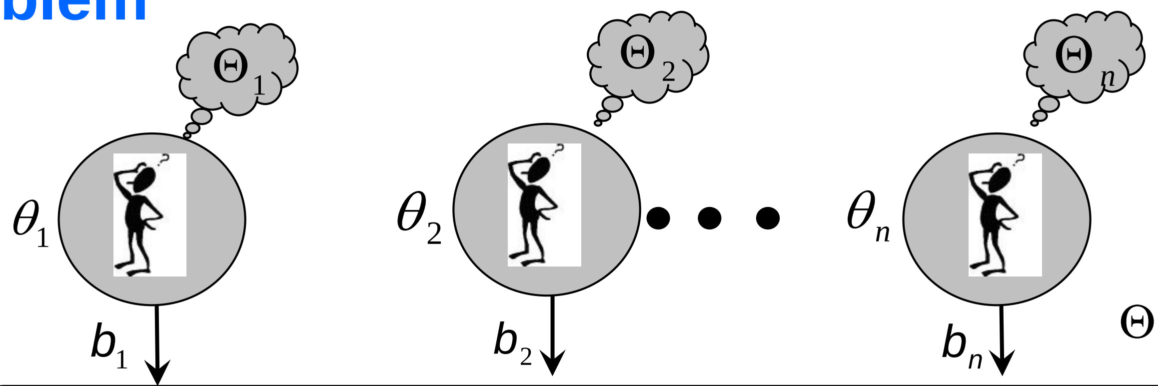
- **Search Engine's View Points**
  - Economic and Computational Performance measures
  - The advertisers' equilibrium bidding strategy profile $\left(s_1^*(.),\cdots,s_n^*(.)\right)$
  - Effect of $\left(s_1^*(.),\cdots,s_n^*(.)\right)$ on performance measures

What is the best Mechanism for Sponsored Search Auction?

- **Economic and Computational Performance Measures**
  - Revenue Maximization
  - Individual Rationality (IR)
  - Incentive Compatibility (IC)
  - Computational Complexity

What is the best Mechanism for Sponsored Search Auction?

- **Sponsored Search Auction as a Mechanism Design Problem**



$$s_i : \Theta_i \rightarrow B_i$$

$$\Theta_i = B_i = \left[\underline{\theta_i}, \overline{\theta_i}\right]$$

$$f(b) = \left(y_{ij}(b),\ p_i(b)\right)_{i \in N,\, j \in M}$$

(Allocation Rule, Payment Rule)

$$u_i\left(f(b), \theta_i\right) = v_i(y(b))\ \left(\theta_i - p_i(b)\right) = \left(\sum_{j=1}^{m} \alpha_j y_{ij}(b)\right) \left(\theta_i - p_i(b)\right)$$

What is the best Mechanism for Sponsored Search Auction?

- **Strategic Bidding Behavior of Advertisers**

If all the advertisers are rational and intelligent and this fact is common knowledge then each advertiser's expected bidding behavior is given by

- **Dominant Strategy Equilibrium (DSE)**

Strategy profile $\left(s_1^*(.),\cdots,s_n^*(.)\right)$ is said to be dominant Strategy equilibrium iff

$$u_i(f(s_i^*(\theta_i),b_{-i})),\theta_i) \geq u_i(f(b_i,b_{-i})),\theta_i) \; \forall b_i \in \Theta_i, \forall b_{-i} \in \Theta_{-i}$$

- **Bayesian Nash Equilibrium (BNE)**

Strategy profile $\left(s_1^*(.),\cdots,s_n^*(.)\right)$ is said to be Bayesian Nash equilibrium iff

$$E_{\theta_{-i}}\left[u_i(f(s_i^*(\theta_i),s_{-i}^*(\theta_{-i})),\theta_i) \mid \theta_i\right] \geq E_{\theta_{-i}}\left[u_i(f(b_i,s_{-i}^*(\theta_{-i})),\theta_i) \mid \theta_i\right] \forall b_i \in \Theta_i$$

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Comparison of OPT with GSP and VCG

- **Incentive Compatibility**

- **VCG:** Follow $s_i^*(\theta_i) = \theta_i$ irrespective of what the others are doing (DSE)
- **OPT:** Follow $s_i^*(\theta_i) = \theta_i$ if all rivals are also doing so (BNE)
- **GSP:** Never follow strategy $s_i^*(\theta_i) = \theta_i$. Use the following BNE strategy

$$s_i^*(\theta_i) = \begin{cases} \theta_i - \dfrac{1}{g(\theta_i,(m-1))} \displaystyle\int_{\theta^l}^{\theta_i} f(x,\theta_i,(m-1))s'(x)dx & : \text{if } n = m \\ \theta_i - \dfrac{1}{g(\theta_i,m)} \displaystyle\int_{\theta^l}^{\theta_i} f(x,\theta_i,m)s'(x)dx & : \text{if } m < n \end{cases}$$

$$f(x,\theta_i,k) = \sum_{j=1}^{k} (j-1)\alpha_j \, {}^{n-1}C_{j-1} (\overline{\Phi}(\theta_i))^{j-2} (\Phi(\theta_i))^{n-j}$$

$$g(\theta_i,k) = k\alpha_k \, {}^{n-1}C_k (\overline{\Phi}(\theta_i))^{k-1} (\Phi(\theta_i))^{n-k-1} + \sum_{j=1}^{k-1} j(\alpha_j - \alpha_{j+1}) \, {}^{n-1}C_j (\overline{\Phi}(\theta_i))^{j-1} (\Phi(\theta_i))^{n-j-1}$$

Comparison of OPT with GSP and VCG

- **Expected Revenue Earned by the Search Engine**

✍ Revenue Equivalence Theorem:

Consider a sponsored search auction setting, in which

1. The advertisers are risk neutral
2. The advertisers are symmetric
3. For each advertiser i, we have $\phi_i(.) > 0$
4. The advertisers draw their types independently

Consider two different mechanisms, each having symmetric and increasing Bayesian Nash equilibrium such that

1. For each possible $(\theta_1, \cdots, \theta_n)$ the final allocation is the same
2. Each advertiser i has same expected utility in two mechanisms for $\theta_i = L$

then equilibria of two mechanisms generate the same expected revenue for the search engine

Comparison of OPT with GSP and VCG

- **Expected Revenue Earned by the Search Engine**

✍ Revenue Equivalence of GSP, VCG, and OPT Mechanisms

Consider a sponsored search auction setting, in which

1. The advertisers are risk neutral
2. The advertisers are symmetric
3. For each advertiser i , we have $\phi_i(.) > 0$
4. The advertisers draw their types independently
5. For each advertiser i , we have $J_i(.) > 0$ and $J_i(.)$ is non-decreasing

Consider three different auction mechanisms – GSP, VCG, and OPT. Let R_{GSP}, R_{VCG} and R_{OPT} be the expected revenue earned by the search engine under these three mechanisms against every query received, then

$$R_{GSP} = R_{VCG} = R_{OPT} \quad \text{if } m < n$$

$$R_{VCG} \le R_{GSP} \le R_{OPT} \quad \text{if } n \le m$$

Comparison of OPT with GSP and VCG

- **Expected Revenue of Search Engine**

<u>Case 1</u> $(m < n)$

$$R_{OPT} = n\left[\int_L^U \left(m\alpha_m \,{}^{n-1}C_m (\overline{\Phi}(x))^m (\Phi(x))^{n-m-1} + \sum_{j=1}^{m-1} j\beta_j \,{}^{n-1}C_j (\overline{\Phi}(x))^j (\Phi(x))^{n-j-1} \right) x\phi(x)dx \right]$$

<u>Case 2</u> $(n \leq m)$

$$R_{OPT} = n\left[\alpha_n L + \int_L^U \left(\sum_{j=1}^{m-1} j\beta_j \,{}^{n-1}C_j (\overline{\Phi}(x))^j (\Phi(x))^{n-j-1} \right) x\phi(x)dx \right]$$

$$R_{VCG} = n\left[\int_L^U \left(\sum_{j=1}^{m-1} j\beta_j \,{}^{n-1}C_j (\overline{\Phi}(x))^j (\Phi(x))^{n-j-1} \right) x\phi(x)dx \right]$$

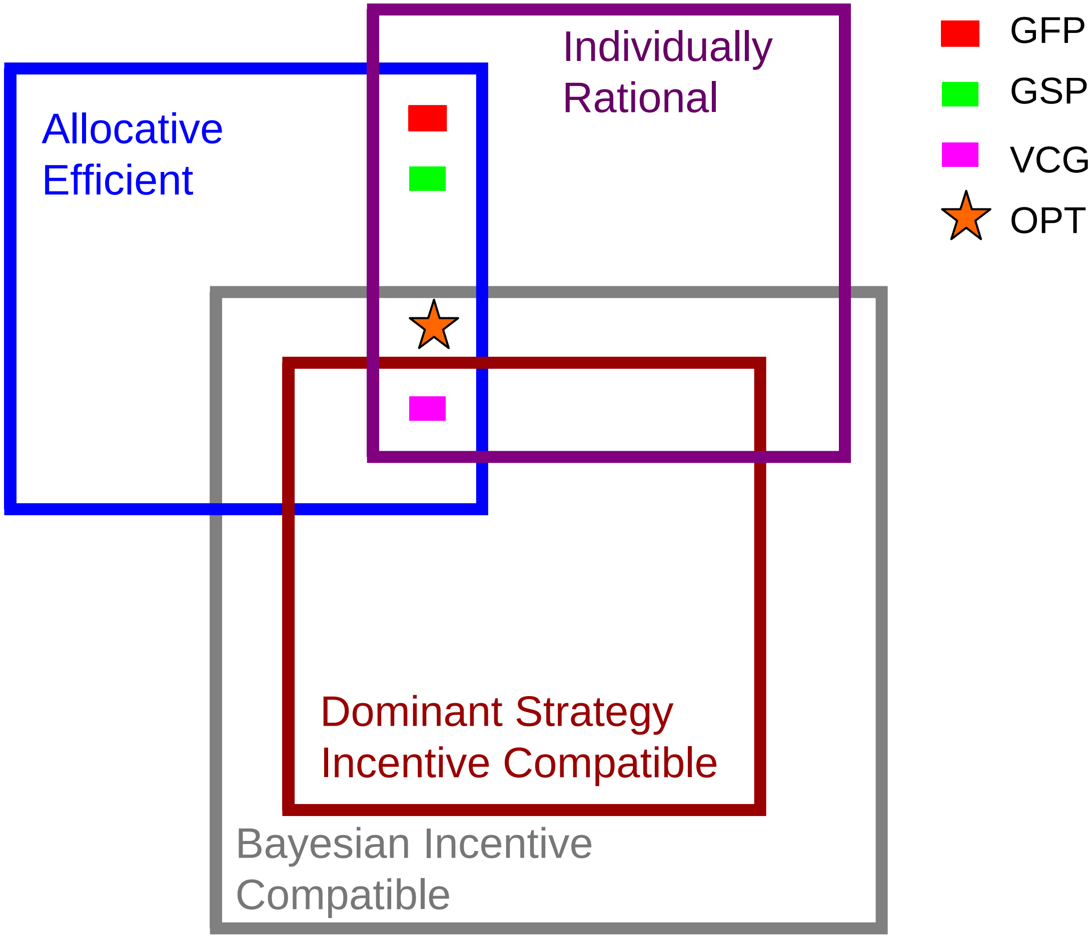
Comparison of OPT with GSP and VCG

- **Economic Performance of Auction Mechanisms**

	Allocation	Payment	DSIC	BIC	IR
GSP	Decreasing order of the bids	Next Highest bid	X	X	✓
VCG	Decreasing order of the bids	Marginal Contribution	✓	✓	✓
OPT	Decreasing order of the bids	Generalized VCG	X	✓	✓

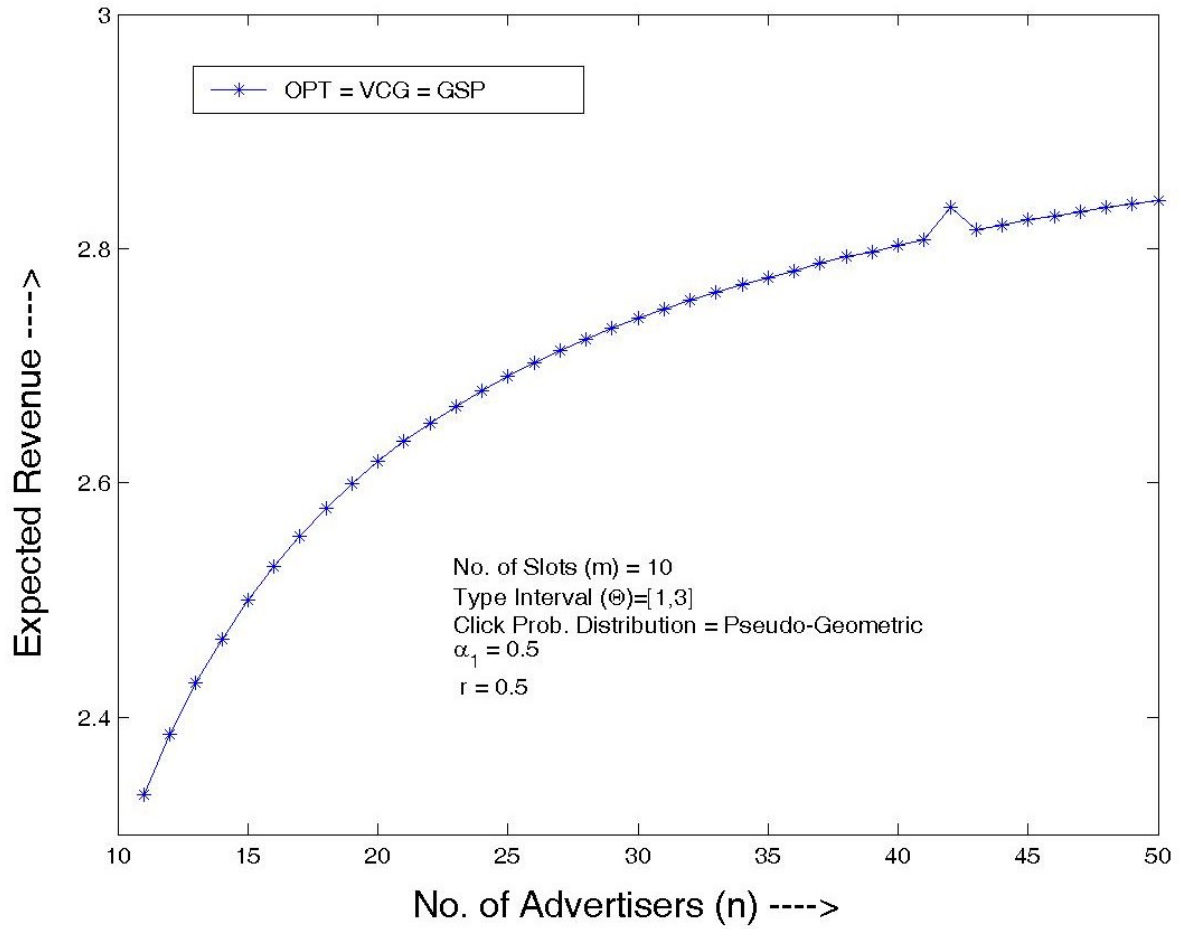
Comparison of OPT with GSP and VCG

- **Economic Performance of Auction Mechanisms**



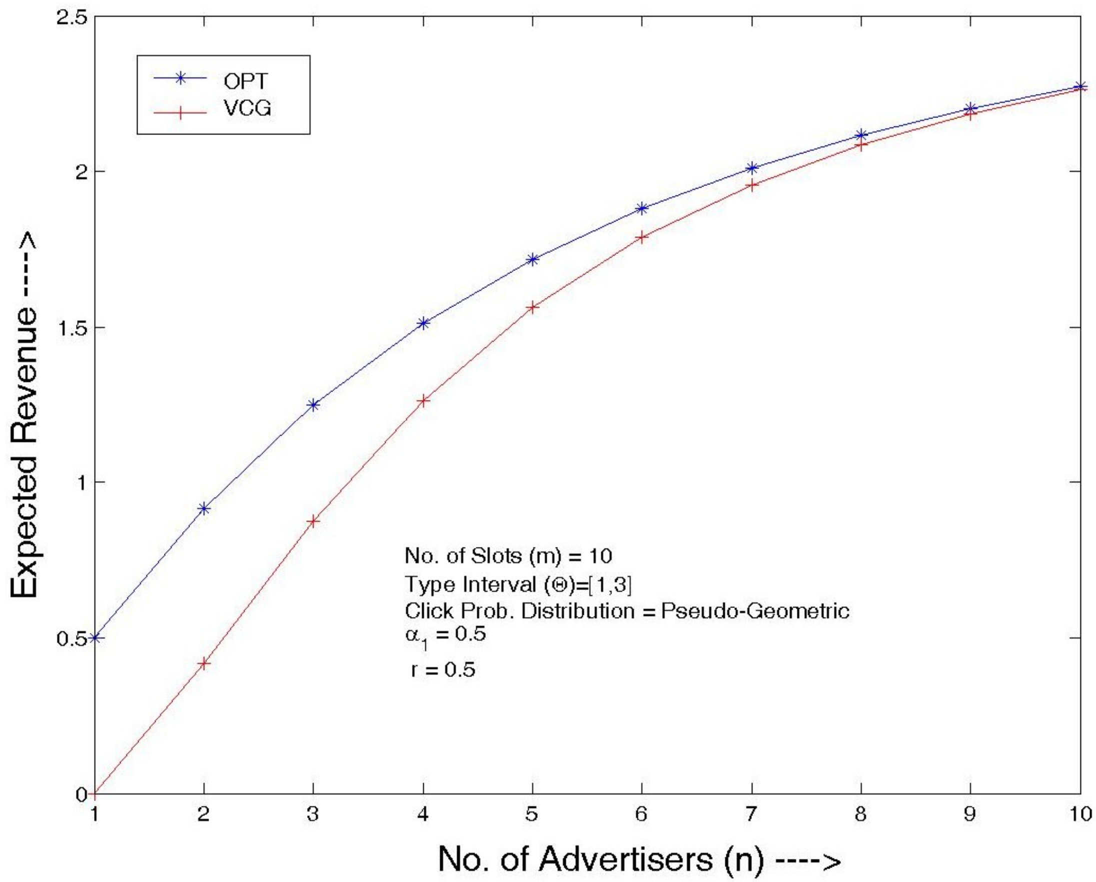
Comparison of OPT with GSP and VCG

- **Experimental Results**



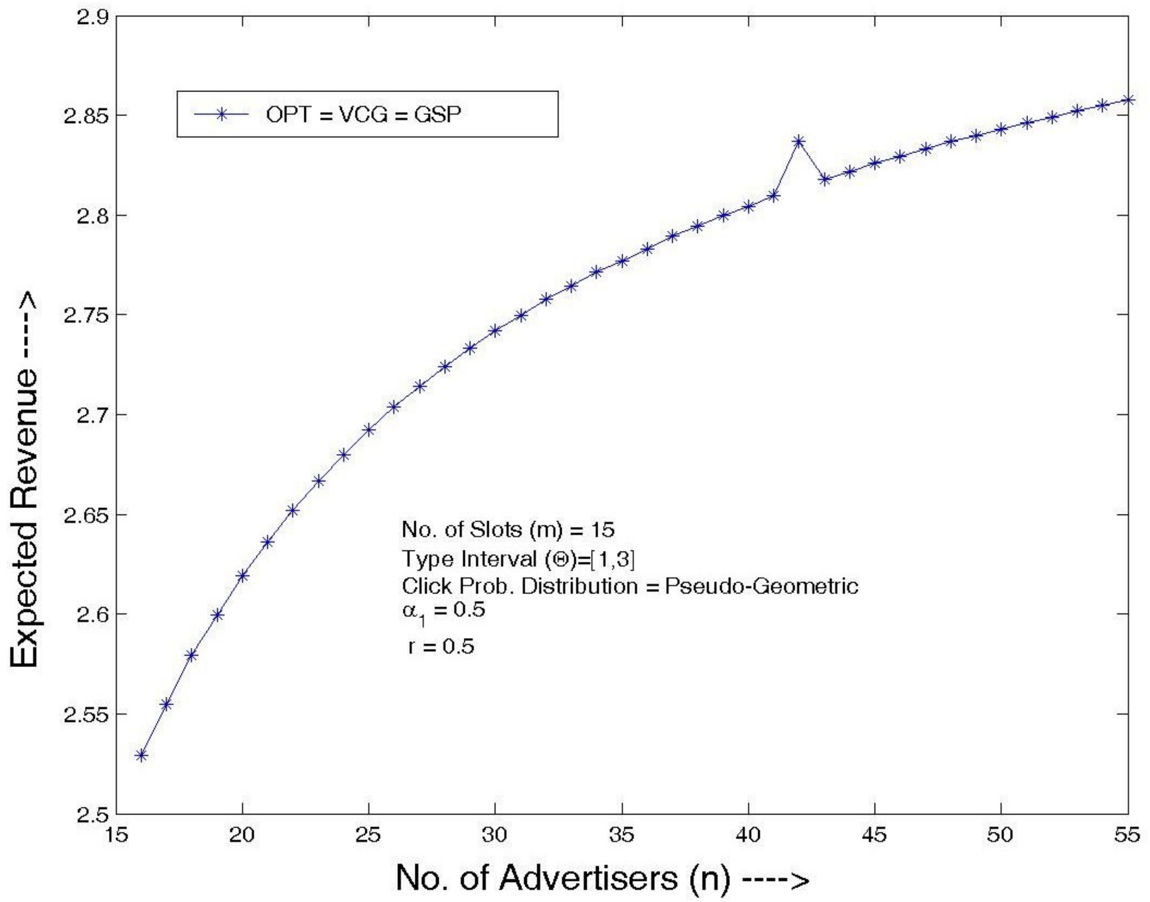
Comparison of OPT with GSP and VCG

- **Experimental Results**



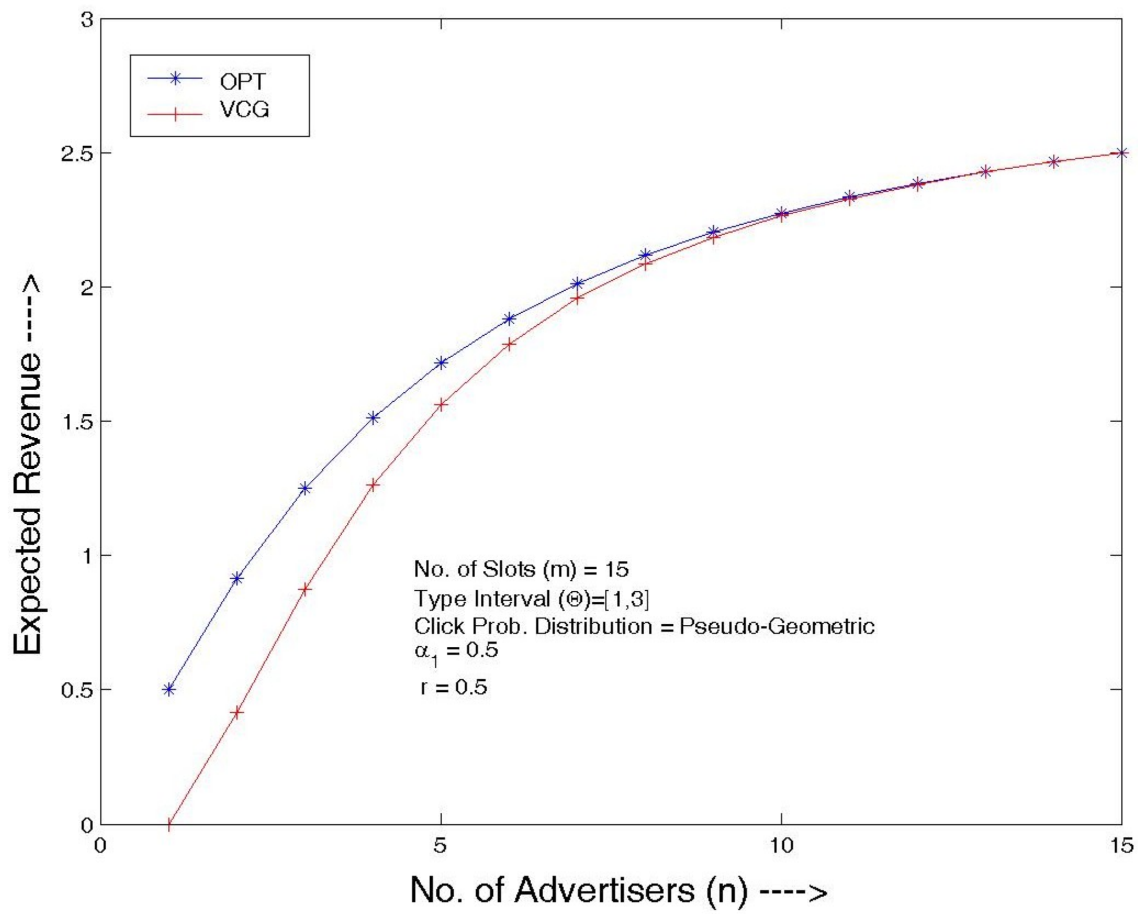
Comparison of OPT with GSP and VCG

- **Experimental Results**



Comparison of OPT with GSP and VCG

- **Experimental Results**



Comparison of OPT with GSP and VCG

- **Computational Performance of Auction Mechanisms**

	Computational Complexity
GSP	$O(n \log n)$
VCG	$O\left(n \log n + (\min(m,n))^2\right)$
OPT	$O\left(n \log n + (\min(m,n))^2\right)$

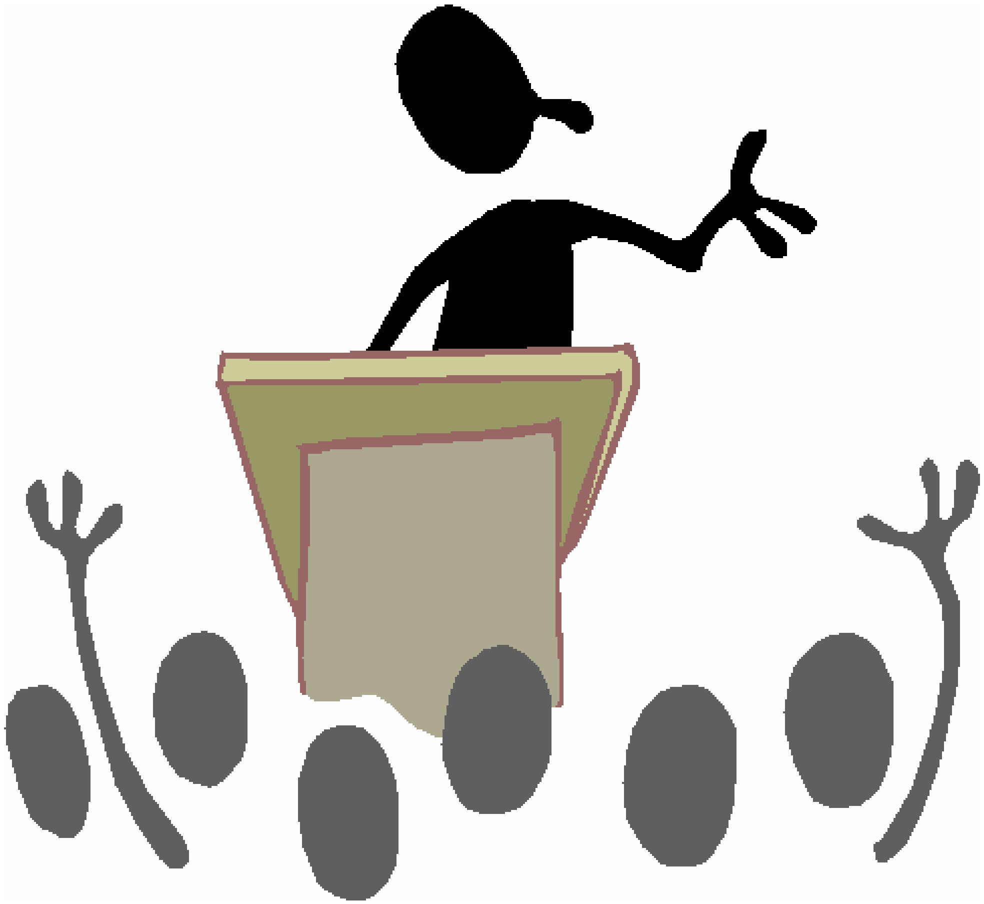
Outline

- ✓ Introduction
  - ✓ Problem Definition
  - ✓ Significance
  - ✓ Related Literature
- ✓ Three well known mechanisms
  - ✓ Generalized First Price (GFP)
  - ✓ Generalized Second Price (GSP)
  - ✓ Vickrey-Clarke-Groves (VCG)
- ✓ A new mechanism – Optimal (OPT)
- ✓ What is the best mechanism for Sponsored Search Auction?
- ✓ Comparison of OPT with GSP and VCG
  - ✓ Incentive Compatibility
  - ✓ Expected Revenue of Search Engine
  - ✓ Individual Rationality
  - ✓ Computational Complexity

Future Directions

- Long Term Goals versus Short Term Goals
- Daily Budget
- Learning the Valuation Distribution $\Phi_i(.)$
- Assumption of Independence of Click Probability on Advertisers' Identity
- Revenue Performance under Asymmetric Advertisers
- Click Fraud
- Competing Search Engines
- Optimal Bidding Strategy of the Advertisers

Questions and Answers …



Thank You …